QP CODE: 19101795

Reg No : .....

Name : .....

B.Sc. DEGREE (CBCS) EXAMINATION, MAY 2019

Second Semester

Core Course - **MM2CRT01 - MATHEMATICS - ANALYTIC GEOMETRY, TRIGONOMETRY AND DIFFERENTIAL CALCULUS**

(Common for B.Sc Computer Applications Model III Triple Main,B.Sc Mathematics Model I,B.Sc Mathematics Model II Computer Science)

2017 ADMISSION ONWARDS

723D5260

Maximum Marks: 80 **Time: 3 Hours**

Part A

Answer any **ten** questions.

Each question carries **2** marks.

1. Derive the condition that the line y = mx + c is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
2. Find the orthoptic locus of the parabola $y^2 = 4ax$.
3. Let P be a point on the circle $x^2 + y^2 = a^2$ and PQ and PR be tangents to the hyperbola $x^2 - y^2 = a^2$. Prove that th elocus of th emiddle point of QR is the curve $(x^2 - y^2)^2 = a^2(x^2 + y^2)$.
4. Show that if a diameter of an ellipse bisects chords parallel to another, then the later will bisect chords parallel to the former.
5. Find the polar coordinates corresponding to the cartesian coordinate $(-3, \sqrt{3})$.
6. Find the polar equation of a circle having the pole lies on the circumference of the circle and the initial line passes through the centre.
7. Prove that $\sin^2 x + \cos^2 x = 1$.
8. Prove that $\cosh 2x = 2(\cosh x)^2 - 1$.
9. Factorize $x^{10} - 1$
10. If $x = f(t), y = g(t)$ prove that $\frac{d^2y}{dx^2} = \frac{f_1 g_2 - f_2 g_1}{{f_1}^2}$ where suffixes denote the differentiation with respect to t.
11. Find the n^{th} derivative of $(ax + b)^m$.
12. Evaluate $lim_{x \to 0} (\cot x)^{\sin 2x}$.

(10×2=20)

Part B

Answer any **six** questions.

Each question carries **5** marks.

13. Find the locus of the point of intersection of two tangents to the parabola $y^2 = 4ax$, which makes an angle α with one another.
14. Show that the chord of contact of tangents from any point on the directrix of a conic passes through the corresponding focus.
15. Show that the locus of the poles of normal chords of $y^2 = 4ax$ is $(x+2a)y^2 + 4a^3 = 0$.
16. If P and Q are extremities of two semi-conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and S is a focus, prove that $PQ^2 - (SP - SQ)^2 = 2b^2$.
17. Show that in a conic the semi latus rectum is the harmonic mean between the segments of a focal chord.
18. Sum to n terms the series $1 + c\cos\alpha + c^2\cos 2\alpha + \ldots$ where c is less than unity.
19. Sum the series $1 + c\cosh\alpha + c^2\cosh 2\alpha + \ldots + c^{n-1}\cosh(n-1)\alpha$., where c is less than unity.
20. If $y = x\log\frac{x-1}{x+1}$, prove that $\frac{d^n y}{dx^n} = (-1)^n(n-2)![\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}]$.
21. Determine $lim[\frac{1}{x^2} - \frac{1}{\sin^2 x}]$ as $x \to 0$.

(6×5=30)

Part C

Answer any **two** questions.

Each question carries **15** marks.

22. If P be any point on the director circle of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, show that the locus of the middle point of th chord in which th epolar of P cuts the ellipse is $(\frac{x^2}{a^2} + \frac{y^2}{b^2})^2 = \frac{x^2+y^2}{a^2+b^2}$.
23. Find the condition in order that the line $\frac{l}{r} = A\cos\theta + B\sin\theta$ may be a tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$.
24. Prove the identities (i) $\cosh 2x = \frac{1+\tanh^2 x}{1-\tanh^2 x}$
    (ii) $\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$
    (iii) $\tanh 3x = \frac{3\tanh x+\tanh^3 x}{1+3\tanh^2 x}$
25. (a) State and prove Leibnitz's theorem to find the n^{th} derivative of the product of two functions.
    (b) Find $y_n(0)$ when $y = [x\sqrt{1+x^2}]^m$.

(2×15=30)